

# GALILEO AND THE PHASES OF VENUS

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## Abstract

In this article we use Galileo's letters and drawings to understand his observations of the phases of Venus in 1610-1611. Our article is presented as an exercise for students to understand the conclusions of the great astronomer.

## PERIODS OF VENUS

- Synodical period and phases

The synodic period is the time that it takes for the object to reappear at the same point in the sky, relative to the Sun, as observed from Earth; i.e. returns to the same elongation and planetary phase. This is the time that elapses between two successive conjunctions with the Sun.

An inferior conjunction occurs when Venus and the Earth lie in a line on the same side of the Sun. When Venus is on the opposite side of the Sun, it is a superior conjunction.

The synodic period of Venus is 584 days. The superior conjunction occurred on 11<sup>th</sup> May 1610. Calculate the date of the quadrature, of the inferior conjunction and of the next superior conjunction, supposing the motions of the Earth and Venus are circular and uniform (Figure 2).

In fact the next superior conjunction occurred on 11<sup>th</sup> December 1611 and inferior conjunction on 26<sup>th</sup> February 1611.

- Sidereal period

The sidereal period is the time that it takes the object to make one full orbit around the Sun, relative to the stars. The sidereal period of the Earth is 365.25 days. Calculate the sidereal period of Venus.

## PHASES OF VENUS IN GEO AND HELIOCENTRIC MODELS

- Phases

1) Determine the phases of Venus in geocentric models, where the Earth is at the center of the universe and planets orbit around (Mercury and Venus "above" or "below" the Sun).

➤ *Plato's model*: Earth (center)-Moon-Sun-Mercury-Venus-Mars-Jupiter-Saturne.

➤ *Ptolemy's model*: Earth (center)-Moon-Mercury-Venus-Sun-Mars-Jupiter-Saturne.

2) Determine the phases of Venus in the heliocentric model, where planets orbit around the Sun.

➤ *Copernican system*: Sun (center)-Mercury-Venus-Earth-Mars-Jupiter-Saturne.

## GALILEO'S OBSERVATIONS

Galileo (1564-1642) observed Venus in 1610-1611 with a telescope. Read his letters. When did Galileo begin to observe Venus? May we conclude that the Copernican model is the only one available? Give the approximate dates of the quadrature and of the inferior conjunction? What are the approximate dates of the five observations of Galileo supposing the Figure 1, from the Assayer, was drawn in 1610-1611?

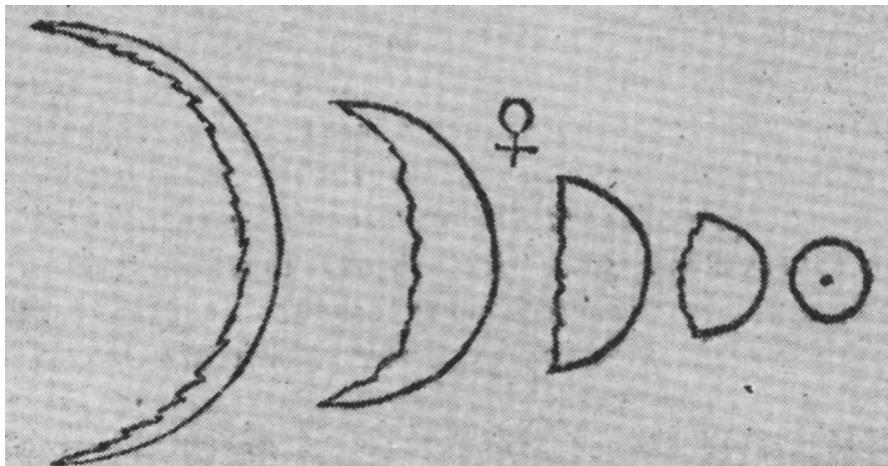


Figure 1. Phases of Venus. Galileo's drawing. The Assayer, 1623

## DISTANCES, ANGULAR DIAMETERS, PHASES OF VENUS

Data:  
 Venus's diameter = 12 100 km  
 Jupiter's diameter = 143 000 km  
 Earth-Sun = 1 a.u. = 150 000 000 km  
 Sun-Venus = 0.723 a.u.  
 Sun-Jupiter = 5.2 a.u.

- Greatest elongation

What is the date of the greatest elongation of Venus (angle  $\beta$  on a 2D drawings Fig. 2)? Why is the elongation not equal to  $0^\circ$  at the inferior conjunction ( $8^\circ$  in 1610)?

- Distances

1) Calculate the distances  $\Delta$  Earth-Venus and the apparent diameters of Venus at the conjunctions  $V_{SC}$  and  $V_{IC}$  and at the greatest elongation  $V_{GE}$ .

2) Compare to the apparent diameter of Jupiter at the opposition.

## DRAWING OF THE TERMINATOR

Calculate the phase (angle  $\epsilon$ ) of Venus on 15<sup>th</sup> September 1610 and draw the phase.

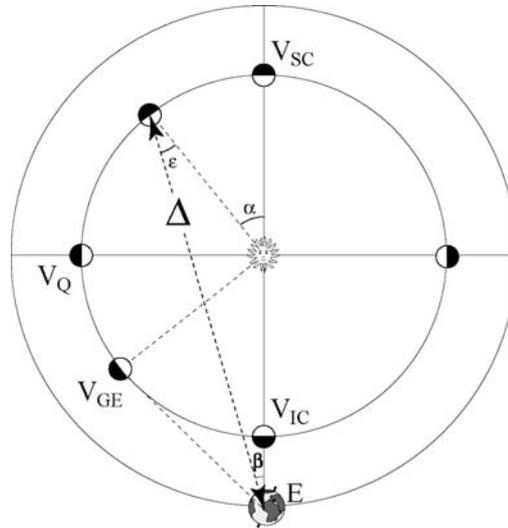


Figure 2. Inferior conjunction  $V_{IC}$ , superior conjunction  $V_{SC}$ , quadrature  $V_Q$  and greatest elongation  $V_{GE}$

## HELIOCENTRIC PLANETARIUM

Place Venus and others planets on 1<sup>st</sup> December 2009 on the heliocentric planetarium. (Figure 3 and documents Reference 3-3).

The inferior conjunction of Venus occurred on 27<sup>th</sup> March 2009.

Calculate the approximate dates of the superior conjunction and of the next inferior conjunction.

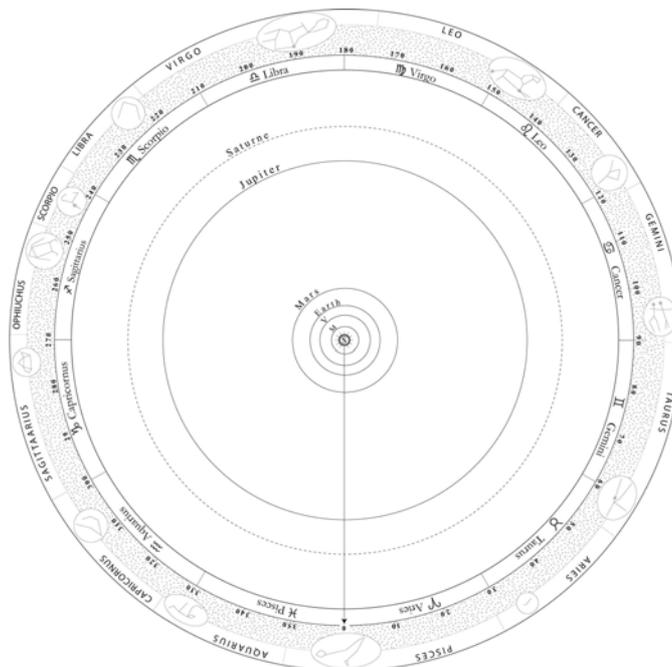


Figure 3. Heliocentric planetarium

Table 1. Planet's heliocentric longitudes in 2010

2010	Dec 09	JAN.	FEB.	MAR.	APR.	MAY	JUN.	JUL.	AUG.	SEPT.	OCT.	NOV.	DEC.
Mercury	297	82	223	302	93	225	315	112	237	331	135	248	343
Venus	226	275	324	8	57	105	156	204	254	303	351	40	88
Earth	69	101	132	161	192	221	251	279	309	339	8	39	69
Mars	103	294	130	143	156	170	183	197	212	227	242	258	275
Jupiter	332	8	337	340	342	345	348	351	354	357	359	2	5
Saturne	178	179	180	181	182	183	184	185	186	187	188	189	190

## EXCERPTS FROM GALILEO'S LETTERS (Stillman Drake's traduction)

- Letter to Giuliano de Medicis, Prague (for Kepler), 11<sup>th</sup> December 1610

*“Haec immatura a me iam frustra leguntur o.y.*

Most illustrious Sir and high Patron

I await desirously the reply to two of mine lately written to your Excellency, to hear what Sig. Kepler shall have said about the oddity of Saturn. Meanwhile I send you the cipher of another particular newly observed by me, which draws in train the decision of great controversies in astronomy, and in particular contains in itself a strong argument for the Pythagorean and Copernican arrangement; and in its time I shall publish the decipherment and other particulars.

I hope I have found the method to determine the periods of the four Medicean planets, deemed with good reason almost insoluble by Sig. Kepler, to whom may it please your Excellency to pay my affectionate respect, as also to Sig. Seggett. From Sig. Hasdale I hear no more, I do not know why. Excuse my brevity, as I do not feel well, and hold me in your grace, for which I am anxious. And with ail reverence I kiss your hand.

From Florence, 11<sup>th</sup> December 1610 (OGG X, 483).”

*Galileo revealed on New Year's Day 1611 that the anagram's signification was “Cynthiae figures aemulatur mater amorum”, i.e. “Shapes of the Moon are imitated by Venus”.*

- Letter to Castelli, Florence. 30<sup>th</sup> December 1610

“To you Reverence's most welcome letter of 5<sup>th</sup> December I shall give a brief reply, being still troubled by an illness that has kept me in bed many days.

I heard with great pleasure your thought of coming to lodge in Florence, renewing my hope to be able again to enjoy and to serve you for some time. Stay fast in this purpose and be sure that I shall always be ready to your convenience, though the felicity of your mind makes it not needful of my works or those of others. As to your questions, I can partly satisfy you, which I do most willingly.

Know then that about three months ago I began to observe Venus with the instrument, and I saw it round in shape and very small; it went daily growing in bulk and keeping the same rotundity until finally, coming to a very great distance from the Sun, it commenced to lose rotundity from the eastern side, and in a few days was reduced to a half-circle. In that shape it stayed many days, but always growing in size; now it begins to become sickle-shaped; and as long as it is seen evenings it will go on thinning its little horns until they vanish. But then returning [in the] mornings, it will be seen with

horns thin and still turned away from the Sun; it will go on growing toward a half-circle until maximum elongation. Then it will remain semicircular for some days, though diminishing in bulk; then from the semi-circle it will pass to a full circle in a few days, and will be seen that way for many months, both as morning and [then as] evening star, all round but very small in size. The very evident consequences drawn from that are well known to your Reverence.

As to Mars, I dare not affirm anything as certain, but observing it for the last four months, it seems to me that in these last days, being in bulk hardly a third of what it was last September, it shows itself somewhat diminished on the east, if my wish does not deceive me, which I do not believe [is the case]. But it will be better seen early next February around quadrature, though by [then] appearing so small, its shape will be hard to distinguish, whether perfectly round or something lacking. But Venus I see as clearly and bounded as the Moon itself, the telescope showing me its diameter equal to the radius of the Moon seen with the naked eye.

Oh, how many consequences, and of what kinds, my dear Benedetto, have I deduced from these and other observations of mine! But what of it? Your Reverence almost made me laugh, saying that the obstinate can be convinced by these observed appearances. You must not know that to convince those capable of reasoning and desirous of learning the truth, other demonstrations previously adduced were enough; but to convince the obstinate who care only for vain applause of the most stupid and stolid herd, testimony by the stars themselves would not suffice, even if they came down to Earth and spoke for themselves. Let us manage to know something for ourselves and rest in that satisfaction only, giving up the wish and hope of advancing in popular opinion, or gaining assent from bookish philosophers ... (OGG X, 502-4)"

- Excerpt from the letter to Cristoforo Clavius, 30<sup>th</sup> December 1610

"... when Venus began to be visible in the evening sky, I started observing it and saw that its figure was circular, though extremely small. Afterwards, I saw [Venus] growing in magnitude significantly, though always maintaining its circular shape. Approaching maximum elongation, [Venus] began to lose its circular shape on the other side from the Sun and within a few days had acquired a semicircular shape. This shape it maintained for a number of days. More precisely, it maintained [this shape] until it began to move towards the Sun, slowly abandoning the tangent. It now begins to assume a notable corniculate shape. Thus, it will continue to decrease during the period in which it remains visible in the evening sky."

- Excerpt from the letter to Paolo Sarpi, 12<sup>th</sup> February 1611

"we are now certain that Venus orbits the Sun, neither [revolving] below (as Ptolemy believed), where it would always show [a phase] less than one half of a circle, nor above (as Aristotle fancied), since if it were above the Sun one would never observe it crescent, but always much more than one half and almost always perfectly circular."

## SOLUTIONS

- Periods of Venus

1) Quadrature ( $\alpha = 90^\circ$ ) 11<sup>th</sup> May 1610 +146 days: 4<sup>th</sup> October 1610.

Inferior conjunction ( $\alpha=180^\circ$ ) 11<sup>th</sup> May 1610 + 292 days: 28<sup>th</sup> February 1611.

Superior conjunction ( $\alpha=360^\circ$ ) 11<sup>th</sup> May 1610 + 584 days: 11<sup>th</sup> May 1611 + 219 = 16<sup>th</sup> December 1611.

In fact the real dates are a bit different (11<sup>th</sup> May 1610 superior conjunction, 26<sup>th</sup> February 1611 inferior conjunction, 11<sup>th</sup> December 1611 superior conjunction) because the Earth and Venus don't move with a circular uniform motion .

2) Sidereal period of Venus

Angular velocities:  $\omega$

$$\omega_{\text{synVenus}} = \omega_{\text{Venus}} - \omega_{\text{Earth}}$$

$$\omega_{\text{Venus}} = \omega_{\text{synVenus}} + \omega_{\text{Earth}} = 2\pi/584 + 2\pi/365.25 = 2\pi/T_{\text{Venus}}$$

$$T_{\text{Venus}} = 365.25 \times 584 / (584 + 365.25) = 224.7 \text{ jours}$$

- Phases of Venus

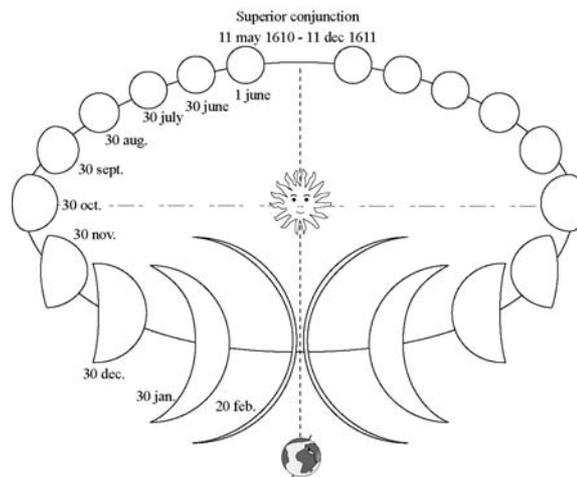


Figure 4. Venus's phases in heliocentric models (according to Palmieri)

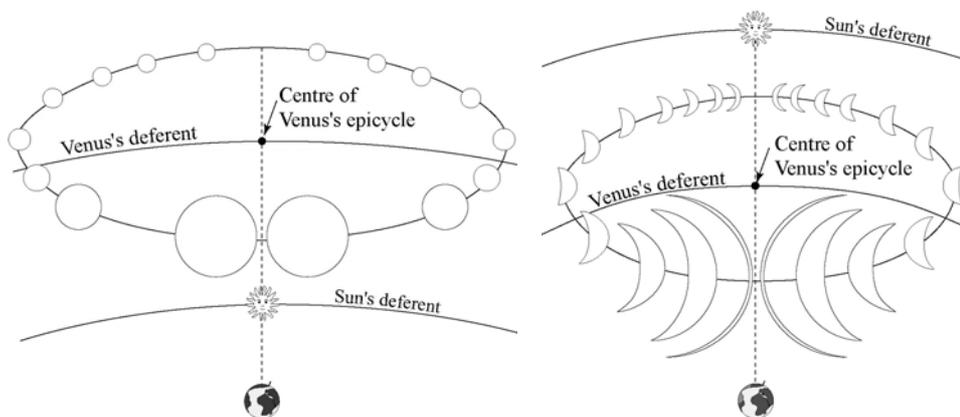


Figure 5. Venus's phases in geocentric models

Galileo concluded from this that Venus moves around the Sun.

Tycho Brahe (1546-1601) advocated an alternative to the Ptolemaic geocentric system, a geo-heliocentric system now known as the Tychonic system. In such a system, the phases of Venus are the same as in the Copernican system.

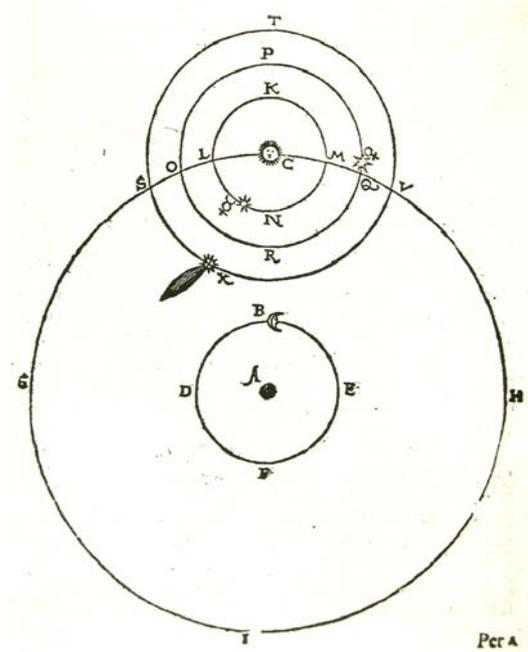


Figure 6. Tycho's solar system model

- Elongations and distances

(TVS) = 90° for the greatest elongation.

$\Delta = \sqrt{1 - 0.724^2} = 0.69$  a.u. and  $\beta = 46.33^\circ$ ,  $\alpha = 136.33^\circ$ ,  $t = 221$  days till the superior conjunction.

Date of the greatest elongation: 18<sup>th</sup> December.

The real greatest elongation occurred on 16<sup>th</sup> December 1610. Our model is a 2D model for a 3D phenomenon, and the real motions are neither circular nor uniform.

Elongation = 8° on inferior conjunction because Venus and Earth don't orbit on the same plane. The elongation is 0° only when a transit occurs (ecliptic latitude of Venus=0°).

	<i>Sup. conjunction</i>	<i>Max elongation</i>	<i>Inf. conjunction</i>	<i>Jupiter opp.</i>
Distances	1.723	0.69	0.277	4.2
Ang. (″ of arc)	9.7″	24.1″	60.1″	46.8″

- Drawing of the terminator

15<sup>th</sup> September = 127 days after the superior conjunction.

$$\alpha = 127 \times 360 / 584 = 78.3^\circ$$

$$(TSV) = 101.7^\circ$$

$$\Delta^2 = SE^2 + SV^2 - 2SE \times SV \cos(TSV) =$$

$$= 1.314 \text{ a.u.}$$

$$\sin(\epsilon/ES) = \sin(180-\alpha)/\Delta$$

Phase angle  $\epsilon = 49^\circ$

We can draw the phase.

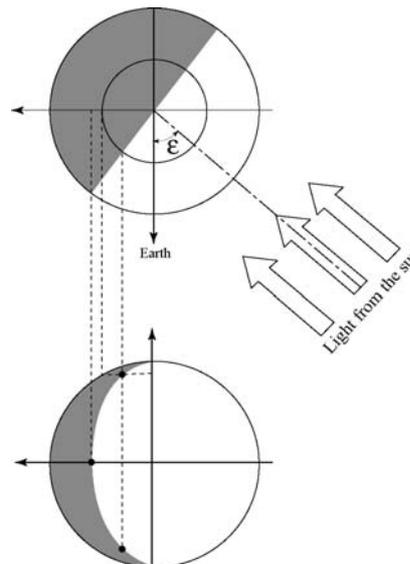


Figure 7. Drawing of the terminator of Venus

## References

- OGG: Opere di Galileo Galilei, ed nazionale, ed by A. Favaro.

### 1) Letters and books of Galileo

- 11<sup>th</sup> December 1610 to Giuliano de Medicis, OGG X 483.
- 1<sup>st</sup> January 1611 to Giuliano de Medici, OGG XI, 11-12.
- 30<sup>th</sup> December 1610 to Benedetto Castelli, (answer to his letter of 22 august 1610) OGG X, 502-504.
- 30<sup>th</sup> December 1610 to Cristoforo Clavius, OGG X 499-502.
- 12<sup>th</sup> February 1611 to Paolo Sarpi, OGG XI, 46-50.
- The Assayer, 1623.
- Dialogue concerning the two chief world systems 1632, OGG VII, 350-..., 366.

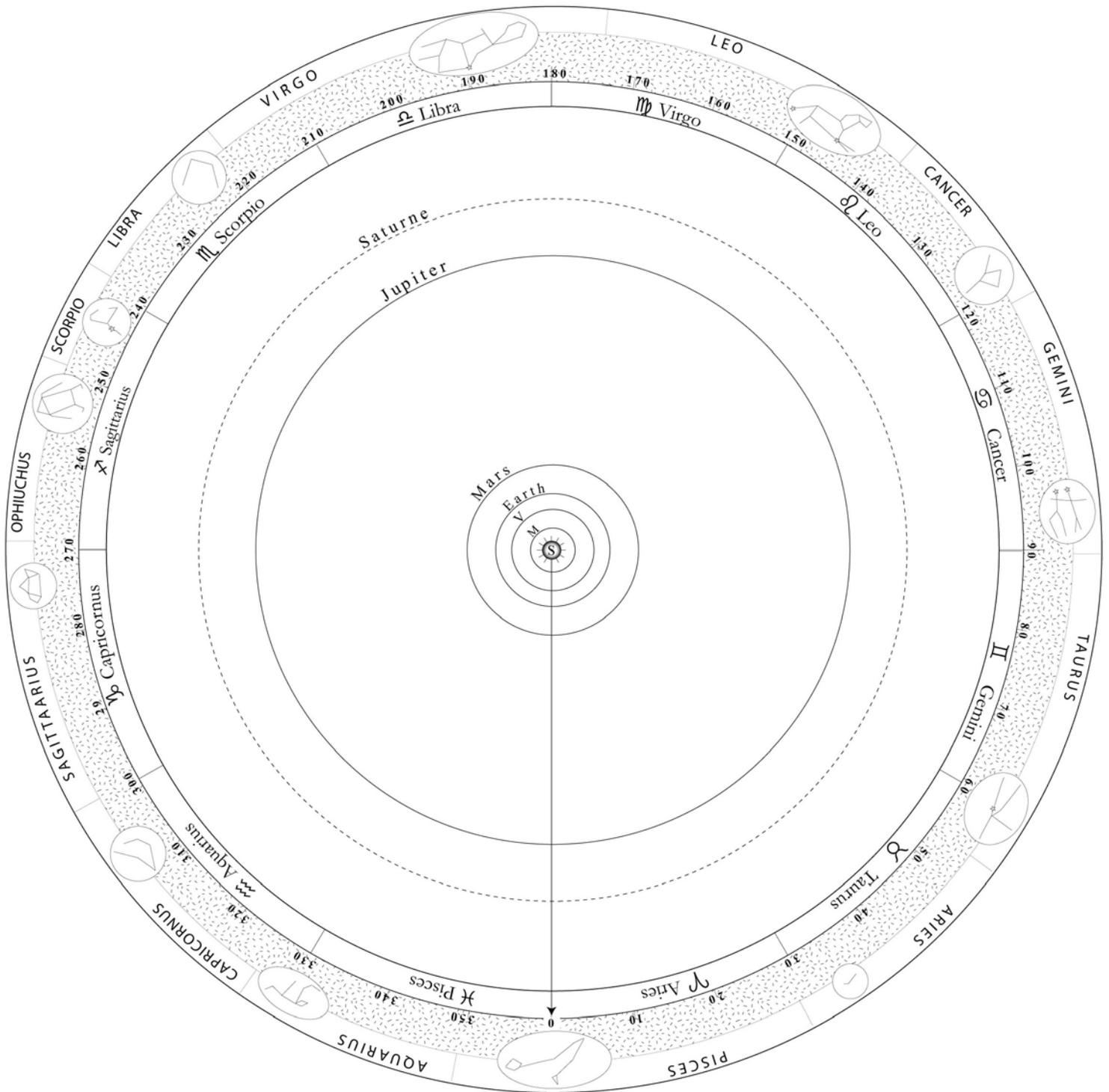
### 2) Articles

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- The appearances of Venus and Mars in 1610, William Peters, JHA, XV 1984.
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- Cahiers Clairaut n°5 et 28 (revue du Comité de Liaison Enseignants Astronomes).

### 3) Ephemerides

- [http://www.imcce.fr/page.php?nav=fr/ephemerides/formulaire/form\\_ephepos.php](http://www.imcce.fr/page.php?nav=fr/ephemerides/formulaire/form_ephepos.php)
- Stellarium: <http://www.stellarium.org/> (choose language at the foot of the home page).
- Documents: [http://isheyvo.ens-lyon.fr/eaee/summerschools/2009/galileo\\_venus/](http://isheyvo.ens-lyon.fr/eaee/summerschools/2009/galileo_venus/)

# Heliocentric Planetarium



	MERCURY	VENUS	EARTH	MARS	JUPITER	SATURNE
Semi-major axis (u.a.)	0,39	0,72	1	1,5	5,2	9,5
Rotation period	88d	225d	365,25d	687d	12years	29,5years
Revolution period	59d	243d (retrograd)	23h56min	24h37min	9h55min	10h14min
Velocity (km/s)	48	35	30	25	13	9
Longitude on 01 12 2009						
Longitude on .. .. .						